PRN: Course Code : USS403

000630 000629

Second Year **B. Sc** End Semester Examination Winter 2024

# Mathematics -IV

Day and Date: Wed 27 NOV 2024 Max Marks: **100**

Time : 02:00PM To 05:00PM Duration: **03** hrs

Instructions: All questions are compulsory.
Sub question 'a)' is compulsory, Solve any two from the remaining

| | | Marks | Bloom's Level | CO |
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| **Q.1** | **Attempt the following** | | | |
| a) | Prove that a non empty subset S of a group $G$ is a subgroup of $G$ if and only if i) $a,b \in S \Rightarrow ab \in S$, ii) $a \in S \Rightarrow a^{-1} \in S$ | 06 | L1 | CO1 |
| b) | Define a subgroup . prove that a non empty subset H of a group is a subgroup of G iff $a,b \in H \Rightarrow ab^{-1} \in H$ | 07 | L3 | CO1 |
| c) | Prove that if $H_1 and H_2$ are two subgroups of group G then $H_1 \cap H_2$ is also a subgroup of G but $H_1 \cup H_2$ is not necessarily a subgroup of G . | 07 | L3 | CO1 |
| d) | Find the order of each element of the group $G = \{1, -1, i, -i\}$ | 07 | L2 | CO1 |
| **Q.2** | **Attempt the following** | | | |
| a) | Define the cyclic group, find how many elements of the cyclic group order 8 . | 06 | L3 | CO2 |
| b) | Prove that the element $a$ and $a^{-1}$ of a group are always of the same order. | 07 | L3 | CO2 |
| c) | Prove that a subgroup of cyclic group is cyclic. | 07 | L3 | CO2 |
| d) | Define a order of an element in a group and Prove that a cyclic group is abelian. | 07 | L1 | CO2 |
| **Q.3** | **Attempt the following** | | | |
| a) | Prove that an infinite cyclic group has exactly two generators. | 06 | L3 | CO3 |
| b) | Find the inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$ | 07 | L2 | CO3 |

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| | c) Show that a subgroup $H$ of a group $G$ is normal in $G$ if and only if $g^{-1}Hg = H$ for all $g \in G$. | 07 | L2 | CO3 |
| | d) Define group homomorphism and given an example. prove that the composition of two homomorphism define on the group is a homomorphism. | 07 | L2 | CO3 |
| **Q.4** | **Attempt the following** | | | |
| | a) Define (i) Field (ii) Subring | 06 | L3 | CO4 |
| | b) A nonempty subset S of a ring R is subring of R if and only if $a.b \in S \Rightarrow a-b \in S$ & $a \cdot b \in S$. | 07 | L2 | CO4 |
| | c) Prove that a field has no proper ideal. | 07 | L2 | CO4 |
| | d) Prove that the intersection of two non-empty subring of a ring is a subring. | 07 | L2 | CO4 |
| Q.5 | Attempt the following | | | |
| | a) Prove that if $f$ is a homomorphism of ring (1) $f(0) = 0^*$ (2) $f(-a) = -f(a)$ | 06 | L2 | CO5 |
| | b) Prove that every ideal is subring | 07 | L2 | CO5 |
| | c) Suppose that R/S is a quotient ring then prove that if R is a commutative then R/S is commutative | 07 | L2 | CO5 |
| | d) Suppose that R/S is a quotient ring then prove that if R has a unity element 1 so also has R/S namely (S+1) | 07 | L2 | CO5 |

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